

IV *A Proposition on the Balance, not taken Notice of by Mechanical Writers, explain'd and confirm'd by an Experiment before the Royal Society, by J. T. DESAGULIERS, L. L. D. F. R. S.*

THO' the following Theorem is agreeable to, and deducible from, Mechanical Principles, yet as it has not been taken Notice of by Mechanical Writers, though often talk'd of among Handicraft Workmen, I thought it might not be improper to take Notice of it here, and to make an Experiment agreeable to the Demonstration.

THEOREM, Figure I.

A B is a Balance, on which is suppos'd to hang at one End B the Scale E with a Man in it, who is counterpois'd by the Weight W hanging at A, the other End of the Balance. I say, that if such a Man, with a Cane or any rigid streight Body, pushes upwards against the Beam any where between the Points C and B (provided he does not push directly against B) he will thereby make himself heavier, or over-poise the Weight W, though the Stop G G hinders the Scale E from being thrust outwards fromwards C towards G G. I say likewise, That if the Scale and Man should hang from D, the Man by pushing upwards against B, or any where between B and D (provided he does not push directly against D) will make himself lighter, or be

be over-poised by the Weight W , which did before only counterpoise the Weight of his Body and the Scale.

If the common Center of Gravity of the Scale E , and the Man supposed to stand in it be at k , and the Man by thrusting against any Part of the Beam, cause the Scale to move outwards so as to carry the said common Center of Gravity to $k \ x$, then instead of BE , LZ will become the Line of Direction of the compound Weight, whose Action will be encreased in the Ratio of LC to BC . This is what has been explain'd by several Writers of Mechanicks; but no one, that I know of, has consider'd the Case when the Scale is kept from flying out, as here by the Post GG , which keeps it in its Place, as if the Strings of the Scale were become inflexible. Now to explain this Case, let us suppose the Length BD of half of the Brachium BC to be equal to 3 Feet, the Line BE to 4 Feet, the Line ED of 5 Feet to be the Direction in which the Man pushes, DF and FE to be respectively equal and parallel to BE and BD , and the whole or absolute Force with which the Man pushes, equal to (or able to raise) 10 Stone. Let the oblique Force ED ($=$ 10 Stone) be resolv'd into the two EF and EB , (or its Equal FD) whose Directions are at right Angles to each other, and whose respective Quantities (or Intensities) are as 6 and 8, because EF and BE are in that Proportion to each other, and to ED . Now since EF is parallel to $BDC A$, the Beam, it does no way affect the Beam to move it upwards; and therefore there is only the Force represented by FD , or 8 Stone to push the Beam upwards at D . For the same Reason, and because Action and Reaction are equal, the Scale will be push'd down at E with the Force of 8 Stone

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also. Now since the Force at E pulls the Beam perpendicularly downwards from the Point B, distant from C the whole Length of the Brachium BD, its Action downwards will not be diminished, but may be express'd by $8 \times \overline{BC}$: Whereas the Action upwards against D will be half lost, by reason of the diminish'd Distance from the Center, and is only to be express'd

by $8 \times \frac{\overline{BC}}{2}$; and when the Action upwards to raise

the Beam is subtracted from the Action downwards to depress it, there will still remain 4 Stone to push down

the Scale; because $\overline{8 \times BC} - 8 \times \frac{\overline{BC}}{2} = 4 \text{ BC}$. Con-

sequently a Weight of 4 Stone must be added at the End A to restore the Æquilibrium. *Therefore a Man, &c. pushing upwards under the Beam between B and D, becomes heavier.* Q. E. D. On the contrary, if the Scale should hang at F from the Point D, only 3 Feet from the Center of Motion C, and a Post *gg* hinders the Scale from being push'd inwards towards C, then if a Man in this Scale F pushes obliquely against B with the oblique Force above-mention'd; the whole Force, for the Reasons before given (in resolving the oblique Force into two others acting in Lines perpendicular to each other) will be reduc'd to 8 Stone, which pushes the Beam directly upwards at B, while the same Force of 8 Stone draws it directly down at D towards F. But as CD is only equal to half of CB, the Force at D compar'd with that at B, loses half its Action, and therefore can only take off the Force of 4 Stone from the Push upwards at B; and consequently the Weight W at A will preponde-

rate, unless an additional Weight of 4 Stone be hang-
ed at B. *Therefore a Man, &c. pushing upwards*
under the Beam between B and D becomes lighter.
Which was also to be demonstrated.

SCHOLIUM I.

Hence knowing the absolute Force of the Man that
pushes upwards, (that is, the whole oblique Force)
the Place of the Point of Trusion D, and the Angle
made by the Direction of the Force with a Perpendi-
cular to the Beam at the same Point, we may have a
general Rule to know what Force is added to the
End of the Beam B in any Inclination of the Direction
of the Force or Place of the Point D.

RULE for the first Case.

First find the perpendicular Force by the following
Analogy, whose Demonstration is known to all that
understand the Application of oblique Forces.

As the Radius:

To the right Sine of the Angle of Inclination ::

So is the oblique Force:

To the perpendicular Force.

Then the perpendicular Force multiplied into the
Length of the Brachium BC, *minus* the said Force
multiplied into the Distance DC, will give the Value
of the additional Force at B, or of the Weight requir'd
to restore the *Æquilibrium* at A.

Or to express it in the Algebraical Way. Let *o f*
express the oblique Force, *p f* the Perpendicular Force,

and x the Force requir'd, or Value of the additional Weight at A to restore the Æquilibrium.

$$DE:DF :: of:pf$$

$$\overbrace{pf \times BC} - \overbrace{pf \times DC} = x$$

The same Rule will serve for the second Case, if the Quantity found be made negative, and the additional Weight suspended at B. Or having found the Value of the Perpendicular Force, the Æquation will stand thus

$-\overbrace{pf \times BC} + \overbrace{pf \times DC} = -x$, and consequently the additional Weight must be hanged at B; because $-x$ at A is the same as $+x$ at B.

SCHOLIUM II.

Hence it follows also, that if, in the first Case, the Point of Trusion be taken at C, the Force at B, (or Force whose Value is requir'd) will be the whole Perpendicular Force; because CD is equal to nothing: And if the Point D be taken beyond C towards A; the Perpendicular Force pushing upwards at that Point, multiplied into DC, must be added to the same Force multiplied into BC, that is $\overbrace{pf \times BC} + \overbrace{pf \times DC} = x$.

The Machine I made use of to prove this experimentally, was as follows. Fig. 2. The Brass Balance AB is 12 Inches long, moveable upon the Center C, with a Perpendicular Piece Bb hanging at the End B, and moveable about a Pin at B, and stopp'd at its lower End b (by the upright Plate GG) from being thrust out of the Perpendicular by the pushing Pipe FE, whose lower Point being put into a little Hole at H, the upper Wire or Point (when put into another little Hole under the Beam at D) is by Means of the

Worm-spring E F pressing against the Plug E to drive forwards the said Wire *b* D, made to push the said Beam upwards with the Force of the Spring. TSS is a Stand, to which is fix'd the Pillar TC that sustains the Balance; and it has also a Slit SS to receive a Shank of the moveable Plate GG, to be fix'd in any Part of the Slit by a Screw underneath.

EXPERIMENT.

Hang on B *b*, as in the Figure. Then let EF be so applied to the Hole H, that its upper Wire *b* D *k* may go through a little Loop at D so as not to thrust the Beam upwards, but be in the same Position as if it did, that by hanging on the Weight W, the Brachium BC with B *b* and FE may be counterpois'd; and then the Action against D and H may be estimated without the Weight of the pushing Pipe.

Then drawing down the End of the Wire *k*, thrust it into the little Hole under D, and B will be so pull'd downwards as to require the additional Weight of 4 Ounces to be hung on at A to restore the Æquilibrium, when BH is 4 Inches, BD 3 Inches, and the whole Force of the Spring equal to 10 Ounces.

I need not here say, that for explaining the second Case, B *b* is to be suspended at D, with the Plate GG fix'd to stop it at the Place M to keep it from being push'd towards T, and that the upper End of GFED *k* must push into an Hole made under B, in which Case the Weight P must be hang'd at B to restore the Æquilibrium.

P. 8. To shew experimentally that the Force which the Spring exerts in this oblique Trusion is equal to 10 Ounces: Take the Beam *A B*, which weighs 4 Ounces, from its Pedestal *C T*, and having suspended at each End, *A* and *B* 3 Ounces, support it under its Center of Gravity by the pushing Pipe *E F* set upright under it, and you will find that the Beam with the two Weights will thrust in the Wire *k b* as far as *b*, the Place which the oblique Trusion drives it to.

Fig. 1.

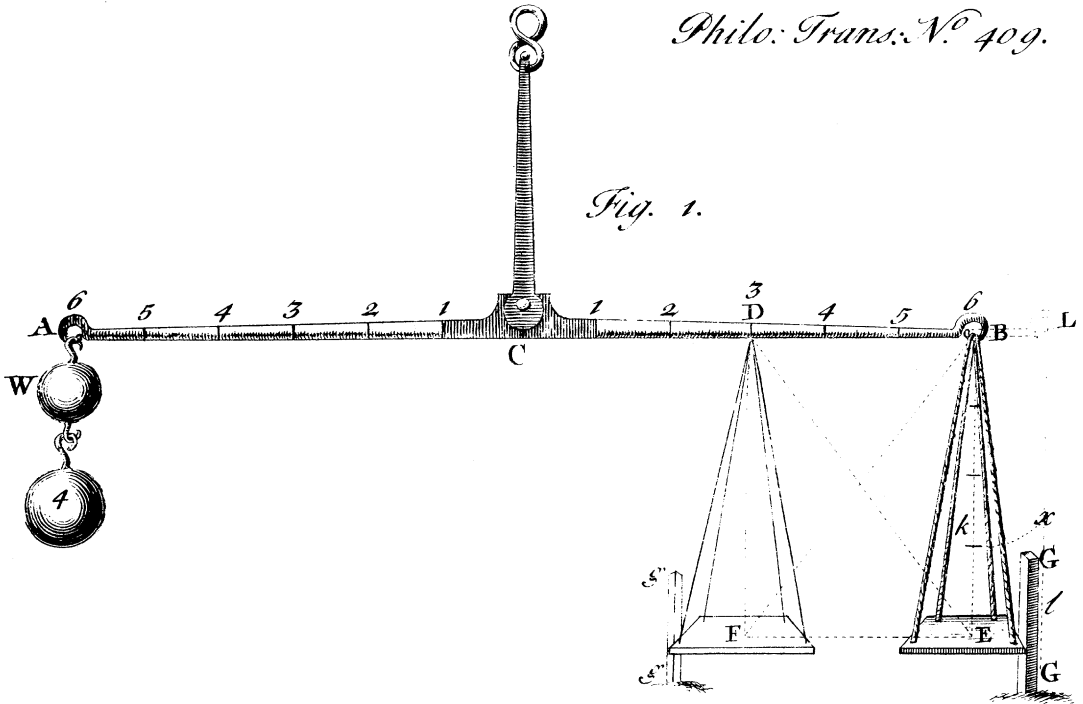


Fig. 2.

